Control Lec 6

Polar Plot

stability

① absolute stability.

② Relative stability

a. Bode diagram
b. Polar Plot
c. Nyquist

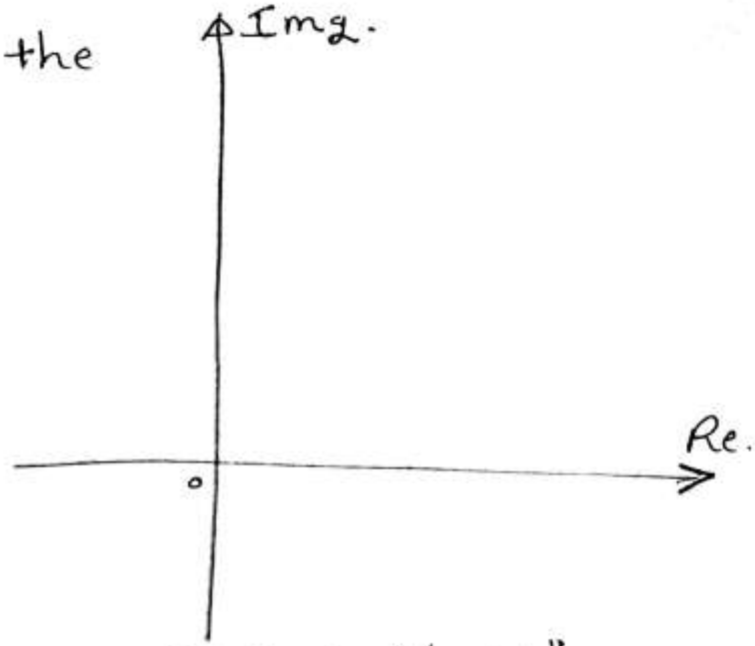
} Freq. analysis

Polar Plot :

→ one of common methods to study Relative stability using freq. analysis technique.

* Polar Plot is a graph for O.L.T.F in Polar Plane which is Consists of two orthognal axis (Real & Imag.) like s-domain.

* Polar plot represent the $|GH|$ and $\angle GH$ in polar plane for different values of ω.



"Polar plane"

steps to draw it

① Map from s-domain to w-domain

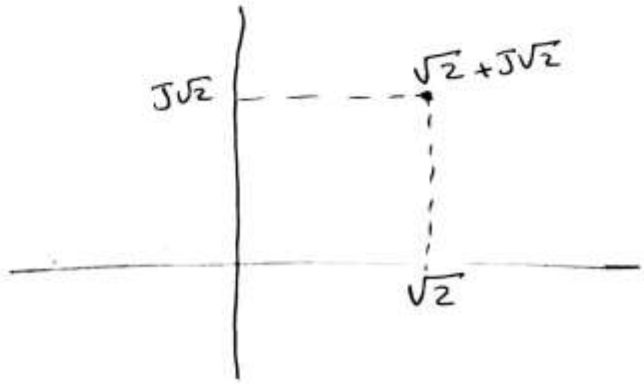
$s \rightarrow J\omega \Rightarrow GH(J\omega) = GH(s)\Big|_{s \Rightarrow J\omega}$

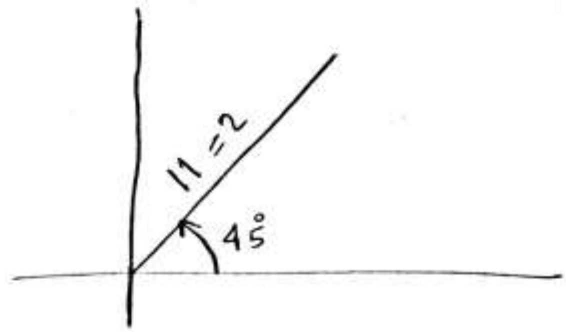
② obtain $|GH(J\omega)|$ and $\phi(\omega) = \angle GH(J\omega)$

③ Draw the $|GH(J\omega)|$ and $\phi(\omega)$ for different values of ω in polar plane

$$* \; GH(s) = \sqrt{2} + J\sqrt{2}$$

$$= \sqrt{2+2} \; \angle \tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = 2 \; \angle 45^\circ$$





ـــ ممكن تعبر عنها بالشكلين.

$\boxed{Ex}$ $$GH(s) = \frac{1}{1 + 0.5\, s}$$

→ Draw the polar plot and find PM & ~~GM~~.

Sol

$$GH(J\omega) = \frac{1}{1 + 0.5\, J\omega}$$

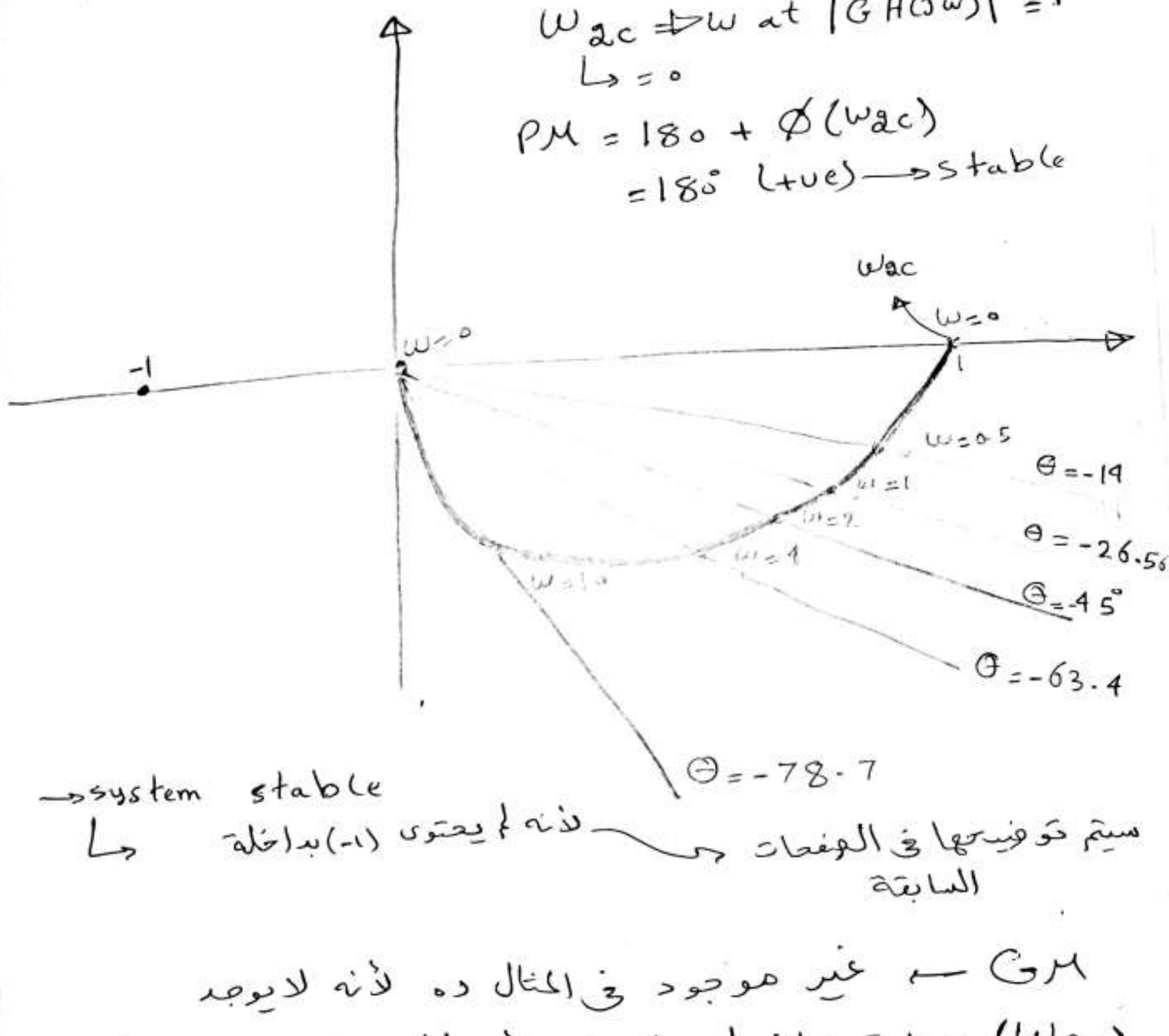
$$|GH(J\omega)| = \frac{1}{\sqrt{1 + (0.5\omega)^2}} =$$

$$\angle GH(J\omega) = 0 - \tan^{-1}(0.5)\omega = \phi(\omega)$$

ω	0	0.5	1	2	4	10	∞
$\lvert GH(J\omega)\rvert$	1	0.97	0.89	$\frac{1}{\sqrt{2}} \downarrow$ 0.707	0.447	$\frac{1}{\sqrt{26}} = 0.2$	0
$\phi(\omega)$	0	-14.04	-26.56	-45	-63.4	-78.7°	-90

$\omega_{gc} \Rightarrow \omega$ at $\lvert GH(J\omega)\rvert = 1$
$\hookrightarrow = 0$

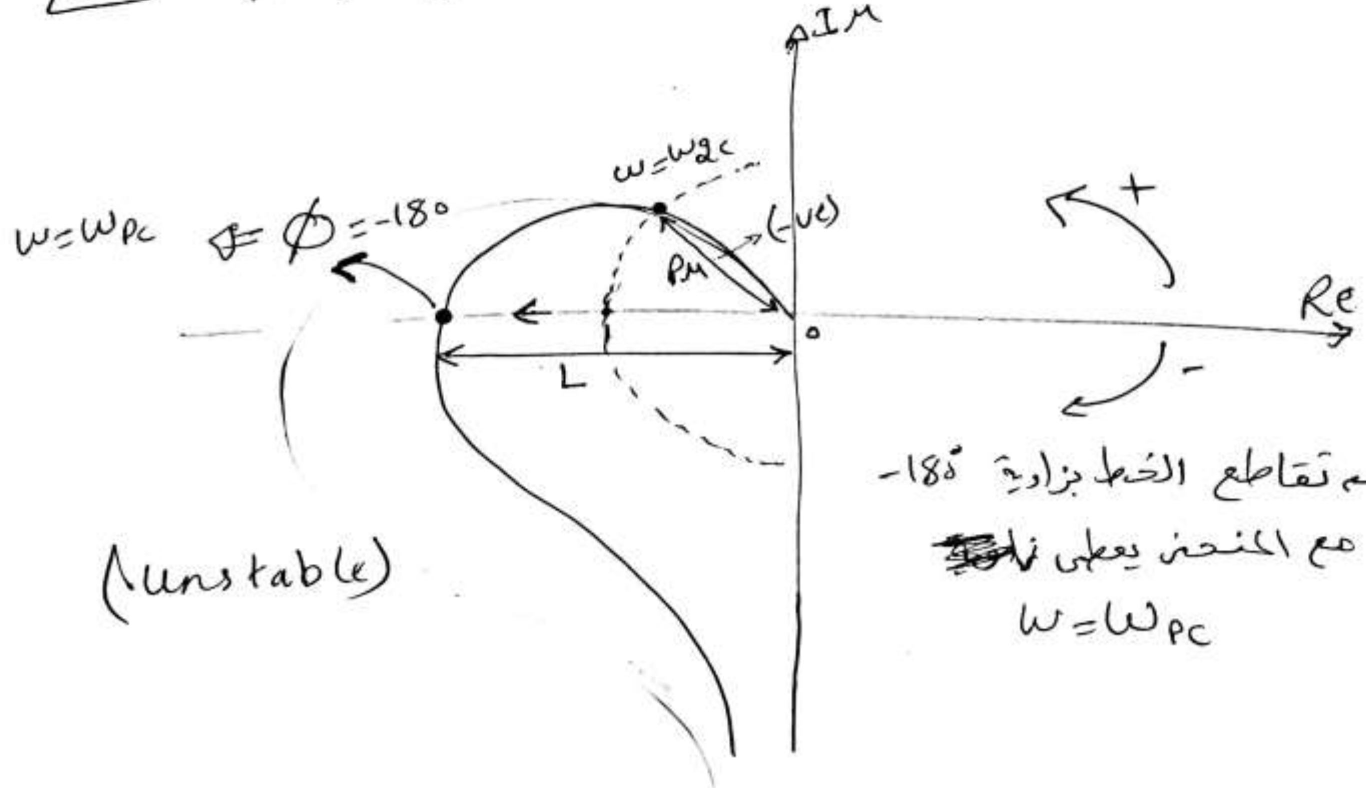
$PM = 180 + \phi(\omega_{gc})$
$= 180^\circ$ (+ve) $\rightarrow$ stable



$\rightarrow$ system stable
$\hookrightarrow$ لأنه لم يحتوى (-1) بداخلة ← سيتم توضيحها فى الصفحات السابقة

GM ← غير موجود فى المثال ده لأنه لا يوجد
(ω_{pc}) هنا توصلنا لـ -180° لأنه أخر حاجة عندنا -90°

$\boxed{EX}$ to find PM & GM



← تقاطع الخط بزاوية -180°
مع المنحنى يعطي
$\omega = \omega_{pc}$

$$PM = 180^\circ + \phi(\underbrace{\omega_{gc}}_{})$$

$$|\,| = 1$$

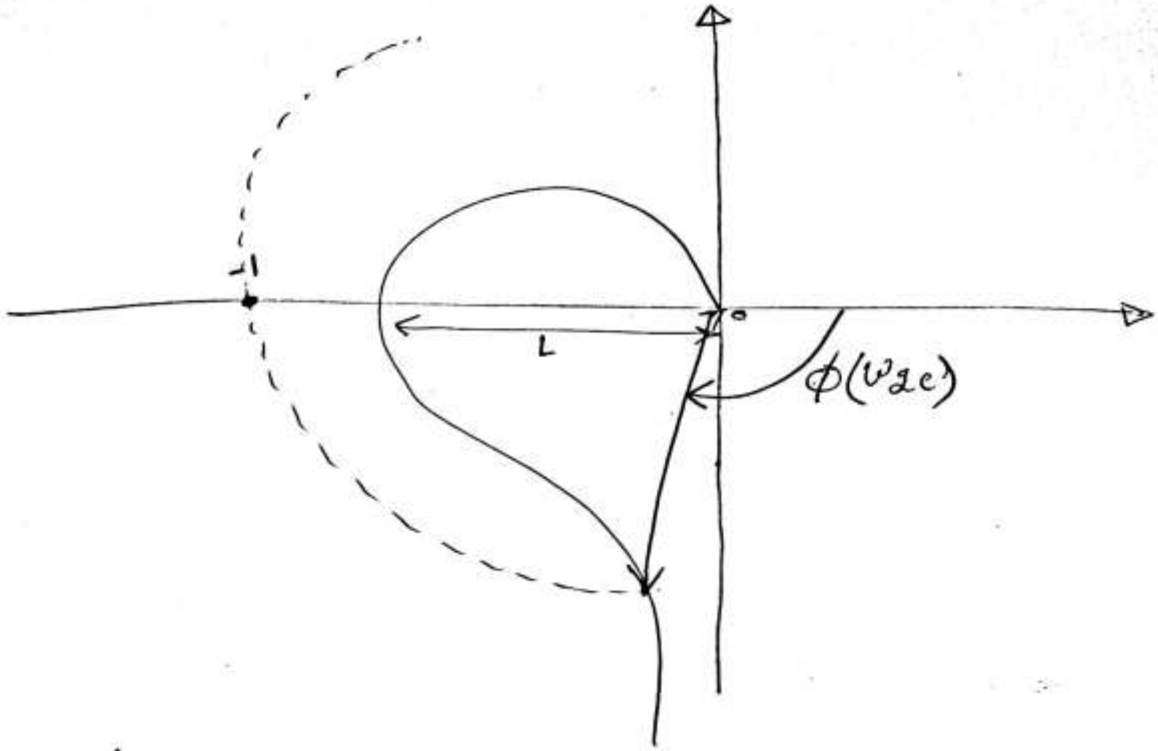
$$|\,|_{dB} = 0$$

← نرسم نصف دائرة نصف قطرها = 1 ← عند الصفر تتقاطع

مع المنحنى في النقطة $\omega = \omega_{gc}$

ال PM ← هو المسافة من الصفر حتى $\omega = \omega_{gc}$

$$GM = \frac{1}{L} \Rightarrow GM\Big|_{dB} = 20 \log\left(\frac{1}{L}\right)$$



$GM = \frac{1}{L} > 1$

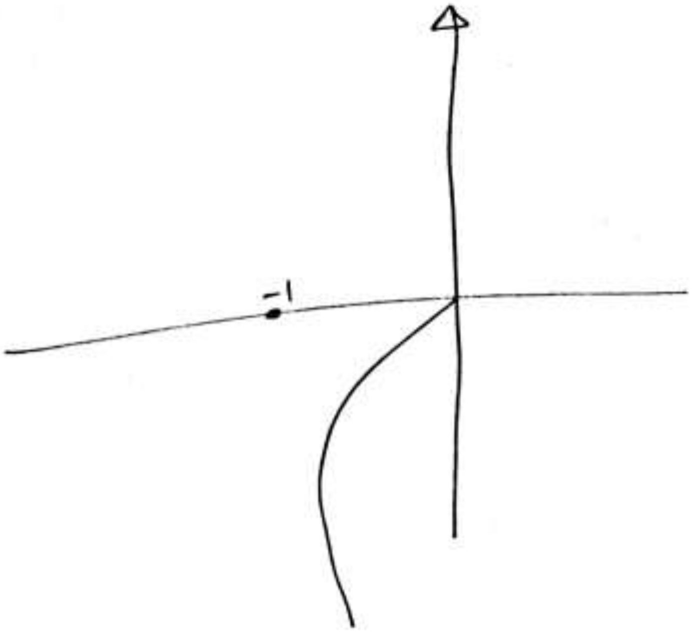
$\longrightarrow$ stable

$PM = 180 + \underbrace{\phi(\omega_{gc})}_{\phi < -180}$

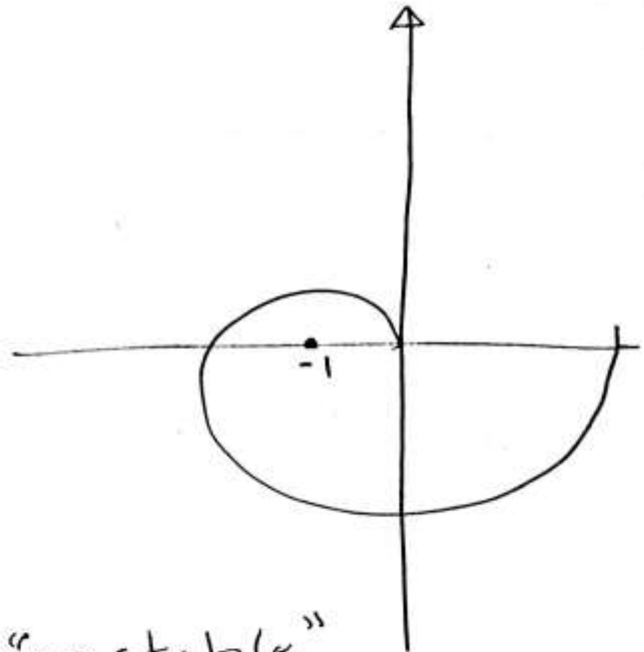
$= +ve$

$\longrightarrow$ system stable

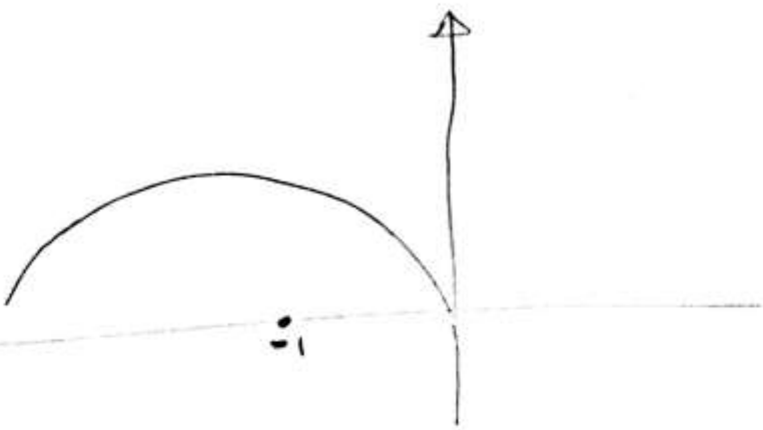
من الصفحة السابقة والصفحة دي نستنتج إن لو المنحنى احتوى (-1) بداخله يكون (unstable) وإذا خرجت -1 بعيد عن المنحنى يكون (stable)



"stable



"unstable"



$\longrightarrow$ unstable

$\boxed{2}$ $$GH(s) = \frac{1}{(2s+1)(0.5s+1)(s+1)}$$

$s \longrightarrow j\omega$

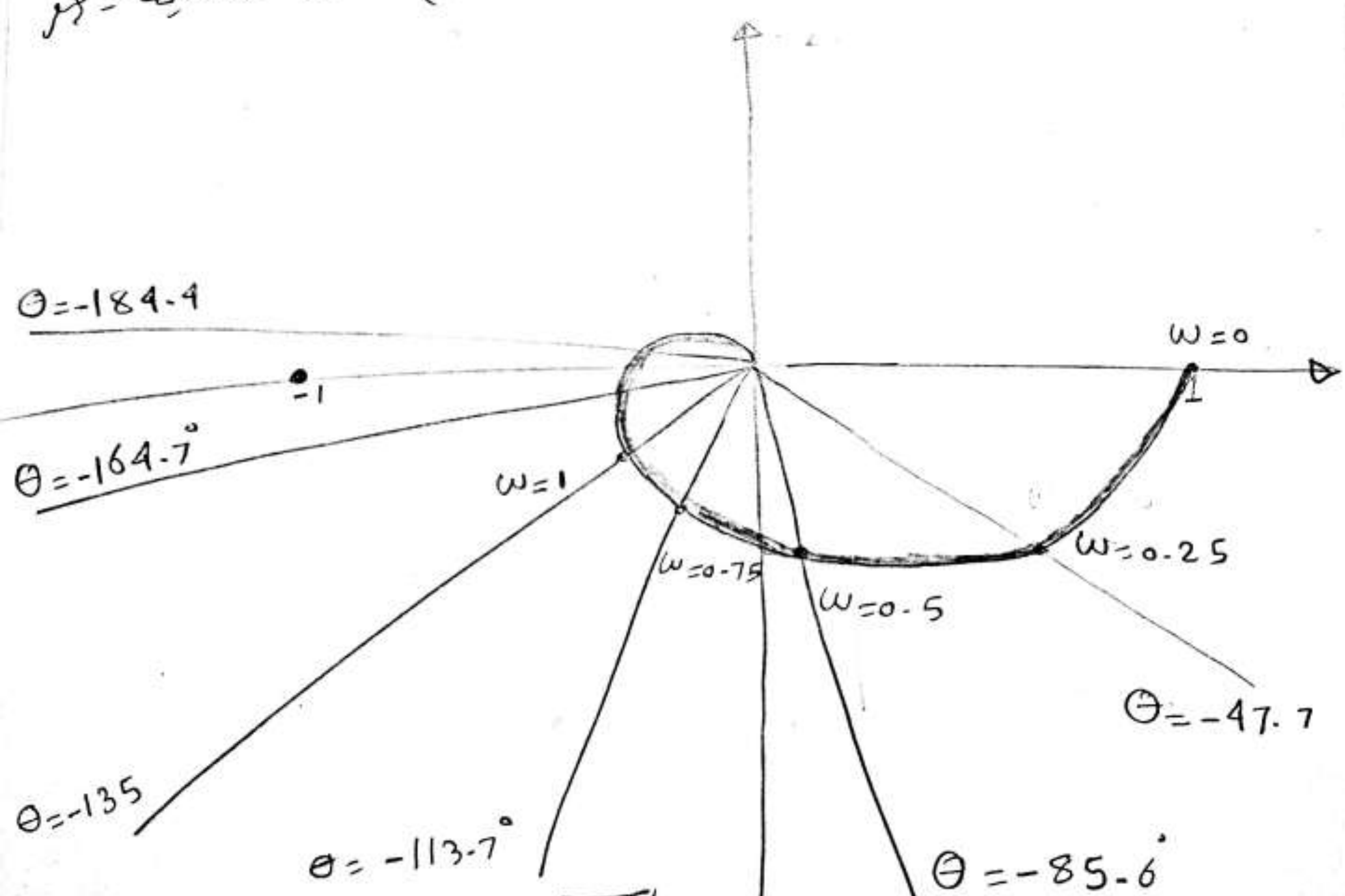
$$GH(j\omega) = \frac{1}{(2j\omega+1)(0.5j\omega+1)(j\omega+1)}$$

$$|GH(j\omega)| = \frac{1}{\sqrt{4\omega^2+1}\ \sqrt{0.25\omega^2+1}\cdot\sqrt{\omega^2+1}}$$

$$\phi(\omega) = -\tan^{-1}(2\omega) - \tan^{-1}(0.5\omega) - \tan^{-1}(\omega)$$

ω	0	0.25	0.5	0.75	1	1.5	2	5	∞
$\lvert GH(j\omega)\rvert$	1	0.86	0.614	0.415	0.28	0.14	0.077	0.072	0
$\phi(\omega)$	0	-47.7°	-85.6°	-113.7°	-135°	-164.7°	-184.4	-231.2	-270

ـــ أي (Physical sys) الـ
(Polar Plot) تنتهي عند القيمة = صفر



→ system stable

PM $PM = 180 + \phi(\omega_{gc})$, $\omega_{gc} = 0$

$= 180 + 0 = 180$ (+ve) → stable

← تبحث عن القيمة اللى عندها ($1 = |GH(j\omega)|$) وتشوف

ω عندها = value (ω_{gc}) ونحسب الزاوية عندها .

GM

$\omega_{pc} = \omega$ at $\phi(\omega) = -180^\circ$

← حل بسيط ← تبحث فى الجدول وتشوف أقرب زاوية ل -180°

و تقلل قيمة ال ω حتى تصل ل -180°

* using try and error (حساب تقريبى)

$\omega = 2 \Rightarrow \phi(\omega) = -184.1^\circ$

$\omega = 1.9 \Rightarrow \phi(\omega) = -181.029^\circ$

$\omega = 1.8 \Rightarrow \phi(\omega) = -177.4^\circ$

$\omega = 1.88 \Rightarrow \phi(\omega) = -180.32^\circ$

$\omega_{pc} \simeq 1.88$ rad/sec

$$GM = \frac{1}{|GH(j\omega)|_{\omega=\omega_{pc}=1.88}}$$

$$GM \simeq 11.37 > 1 \quad (\text{stable})$$

<u>OR</u>

$$GM_{dB} = 20\log GM = 20\log(11.37)$$
$$= 21.16\ dB$$

$$GM \rightarrow +ve \quad (\text{stable})$$

← لو عايز أحسب ω_{pc} بحل رياضي

$$\phi(\omega) = -\tan^{-1}(2\omega) - \tan^{-1}(0.5\omega) - \tan^{-1}(\omega)$$

$$\omega_{pc} \rightarrow \omega \quad \text{at} \quad \phi(\omega) = -180$$

$$-180 = -\tan^{-1}(2\omega) - \tan^{-1}(0.5\omega) - \tan^{-1}(\omega)$$

$$\tan(x \pm y) = \frac{\tan(x) \pm \tan(y)}{1 \mp \tan x \cdot \tan y}$$

$$180 - \tan^{-1}(2\omega_{pc}) = \tan^{-1}(0.5\omega_{pc}) + \tan^{-1}(\omega_{pc})$$

ـــه ناخذ tan للطرفين

$$\frac{\tan(180) - 2\omega_{pc}}{1 + \tan(180) \cdot (2\omega_{pc})} = \frac{0.5\,\omega_{pc} + \omega_{pc}}{1 - 0.5\,\omega_{pc}^2}$$

$$-2\cancel{\omega_{pc}} = \frac{1.5\,\cancel{\omega_{pc}} \cancel{+ \omega_{pc}}}{1 - 0.5\,\omega_{pc}^2}$$

$$1.5 = -2 + \omega_{pc}^2 \Rightarrow \omega_{pc}^2 = 3.5$$

$$\omega_{pc} = \sqrt{3.5} = 1.8708 \text{ rad/sec}$$

ـــه تحسب منها GM.

$\boxed{3}$ $$G.H(s) = \frac{1}{s(1+s)(0.5+s)}$$

type 1 sys.

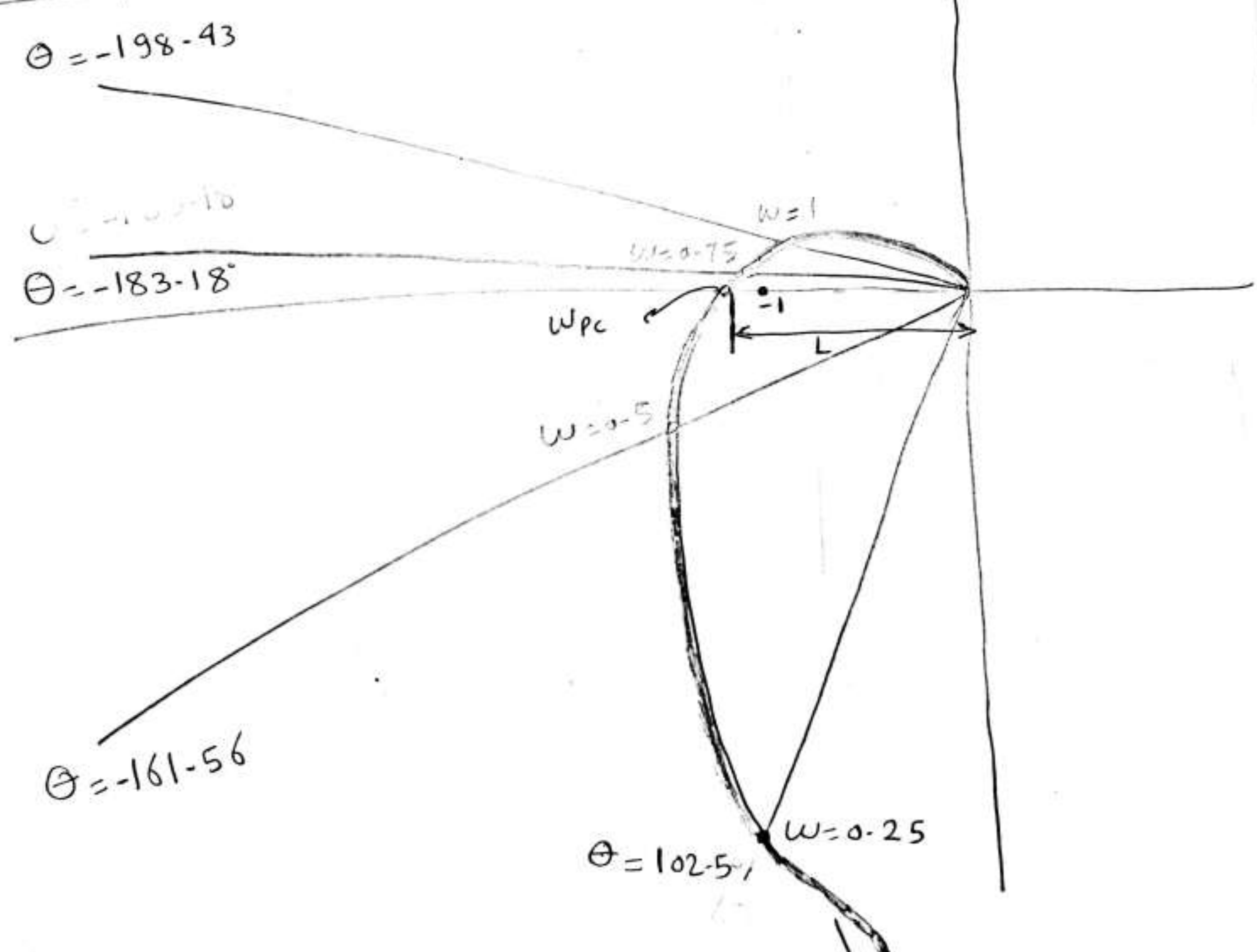
$s \rightarrow j\omega$

$$GH(j\omega) = \frac{1}{j\omega(1+j\omega)(0.5+j\omega)}$$

$$|GH(j\omega)| = \frac{1}{\omega\sqrt{1+\omega^2} * \sqrt{0.25+\omega^2}}$$

$$\phi(\omega) = -90^\circ - \tan^{-1}(\omega) - \tan^{-1}(2\omega)$$

ω	0	0.25	0.5	0.75	1	1.5	2	∞
$\lvert GH(j\omega)\rvert$	∞	6.9	2.53	1.183	0.6	0.234	0.108	0
$\phi(\omega)$	-90°	-102.5	-161.56	-183.18°	-198.43	-217°	-229.4°	-270°



$GM = \frac{1}{L} < 1$ un stable

<u>or</u>

$\omega_{pc} \Rightarrow \omega$ at $\phi = -180^\circ$

$\omega = 0.75 \Rightarrow \phi(\omega) = -183.18^\circ$

$\omega = 0.7 \Rightarrow \phi(\omega) = -179.4^\circ$

$\omega = 0.72 \Rightarrow \phi(\omega) = -180.97^\circ$

$\omega = 0.71 \Rightarrow \phi(\omega) = -180.22^\circ$

$\omega = 0.708 \Rightarrow \cancel{\omega} \boxed{\phi(\omega) = -180.068^\circ}$

$\omega_{pc} \simeq 0.708$ rad/sec

$$GM = \frac{1}{|GH|_{\omega = \omega_{pc}}} = 0.752 < 1 \rightarrow \text{unstable}$$

$\omega_{gc} \Rightarrow \omega$ at $|GH| = 1$

$\omega = 0.75 \Rightarrow |\ | = 1.183$

$\omega = 0.8 \Rightarrow |\ | = 1.034$

$\omega = 0.81 \Rightarrow |\ | = \boxed{1.0078}$

$\omega_{gc} \simeq 0.81$ rad/sec

$PM = 180^\circ + \phi\,(\omega_{gc} = 0.81)$

$= -7.32^\circ < 0$ (unstable)

← في ال (mid term) هيجي سؤال لل (Polar plot)

مش هيجي في الفاينال.